International Research Training Group IGDK 1754

IGDK Students' Workshop

Parameter Estimation Problems in Physically Based Image Processing

Lukas Failer and Philipp Jarde

October 4.-9., 2015

1 Motivation

To get additional insight about the cardiovascular condition of a patient, scientist in [1, 2, 8] aim to simulate the behavior of the patient's aorta. To adapt the model of the aorta automatically to the patient, parameter estimation methods can be applied to determine the missing patient specific data. But in medical applications quite often the only measurement data available, to compare simulation results with, are MR-Images as in Figure 1. Therefore in addition to the parameter estimation problem we have to solve an optical flow problem first to determine the displacement field U, describing the deformation of the aortic wall.

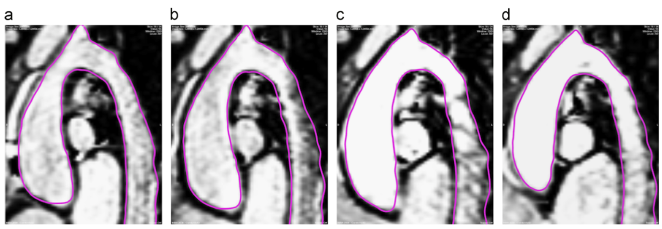

Figure 1: MR-Image of aorta at 4 time points taken from [1]

If we assume that the total time derivative of the brightness intensity $I(x,t)$ at each temporal and spatial point is zero the optical flow equation

$$\partial_t I + b \cdot \nabla I = 0 \quad \text{in } \Omega \times (0,T)$$

has to be fulfilled. To calculate the optical flow $b(x,t)$ the authors in [4, 3] suggest to solve the following optimal control problem

$$\min J(b,I) := \frac{1}{2} \sum_{i=1}^{N} \|I(t_k) - I_k\|^2_{L^2(\Omega)} + R(b)$$

subject to

$$\begin{aligned} \partial_t I + b \cdot \nabla I &= 0 \quad \text{in } \Omega \times (0,T), \\ I(0) &= I_0 \quad \text{in } \Omega. \end{aligned}$$

Thereby $R(b)$ is a suitable regularization and I_k is the brightness intensity of the given sample images. For the given optical flow we can reconstruct the displacement field U by transforming the velocity field b in Lagrange coordinates:

$$\partial_t U = b(t, x+U) \quad \text{in } \Omega \times (0,T).$$

For such a given displacement field U a parameter estimation problem can be formulated to determine patient specific material parameters of the aorta. We have to solve the following optimal control problem as presented for example in [8].

$$\min J(q,u,v) := \frac{1}{2}\int_0^T \|u - U\|^2_{L^2(\Omega_s)}\,\mathrm{d}t + \frac{\alpha}{2}\|q\|^2_Q$$

subject to

$$\begin{aligned} v_t - \operatorname{div}(\sigma_f(v,p)) + (v \cdot \nabla)v &= 0 && \text{in } \Omega_f(u) \\ \operatorname{div}(v) &= 0 && \text{in } \Omega_f(u) \\ u_{tt} - \operatorname{div}(\Sigma_s(q,u)) &= 0 && \text{in } \Omega_s \end{aligned}$$

The fluid velocity v is described by a Navier-Stokes equation and the aortic wall deformation is modeled by a nonlinear stress tensor Σ whereby the stress tensor depends on an unknown parameter q. Some ideas how to solve the given optimal control problem can be found in [5, 9, 8, 2].

2 Physically based optical flow

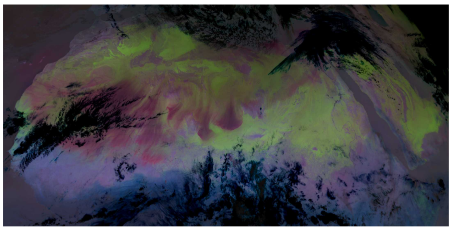

Figure 2: False colour image taken by a geostationary satellite. Sand dust plumes are colored in magenta taken from [7]

For parameter estimation problems with given image data the question arises if better results can be obtained, if we use an optical flow algorithm which reproduces the underlying dynamic. M. Klinger developed in his Phd thesis [7] such a physically based optical flow formulation due to tackling the optical flow and parameter estimation problem at once. To determine boundary conditions for the fluid flow for a given brightness intensity of a transported visible substance (see Figure 2) he analyzed the following optimal control problem:

$$\min J(q,u,v) := \frac{1}{2}\sum_{k=1}^{N} \|I(t_k) - I_k\|^2_{L^2(\Omega)} + \frac{\alpha}{2}\|q\|^2_Q$$

subject to

$$\begin{aligned}
\partial_t I - \varepsilon I + u \cdot \nabla I &= 0 && \text{in } \Omega \times (0,T)\\
\partial_t u - \nu \Delta u + u \cdot \nabla u + \nabla p &= 0 && \text{in } \Omega \times (0,T)\\
\nabla \cdot u &= 0 && \text{in } \Omega \times (0,T)\\
u &= q && \text{on } \Gamma_q \times (0,T)\\
u &= 0 && \text{on } \Gamma_I \times (0,T).
\end{aligned}$$

3 Physically based optical flow for the wave equation

The changes in the images of the aorta in Figure 1 are caused by the displacement of the aortic wall. The displacement field fulfills a nonlinear wave equation. Motivated by the introductory example we would like to adapt in the project " Parameter Estimation Problems in Physically Based Image Processing " the methods presented in [7] to configurations subjected by a linear wave equation. Such an approach would lead to

$$\min J(q,I) := \frac{1}{2}\sum_{i=1}^{N} \|I(t_k) - I_k\|^2_{L^2(\Omega)} + \frac{\alpha}{2}\|q\|^2_Q$$

subject to

$$\begin{aligned}
\partial_t I + \partial_t u(t, x-u) \cdot \nabla I &= 0 && \text{in } \Omega \times (0,T),\\
\Delta u &= 0 && \text{in } \Omega/\Omega_s \times (0,T),\\
\partial_{tt} u - \nu \Delta u &= f + Bq && \text{in } \Omega_s \times (0,T).
\end{aligned}$$

4 Outline

In the project we want to discuss if the given optimal control problem is well posed and if not how we have to change the optimal control problem. In addition we have to consider if we have to adapt the optimal flow equation as in [7]. For additional information on physically based image processing see [6, 10]. The focus of the project should be on discretizing the corresponding optimality system and to test the numerical algorithm for a simple example. As the sample images are only given at a discrete number of time points it can be reasonable to solve the optimal control problem on each subinterval separately. To validate the algorithm we plan to compare the results with an approach solving parameter estimation and optical flow problems separately. A possible test configuration could be an oscillating beam fixed at one side and a parameter q controlling the volume force.

5 Bibliography

References

[1] C. Bertoglio, D. Barber, N. Gaddum, I. Valverde, M. Rutten, P. Beerbaum, P. Moireau, R. Hose, and J. Gerbeau, *Identification of artery wall stiffness: In vitro validation and in vivo results of data assimilation procedure applied to a 3d fluid-structure interaction model*, Journal of Biomechanics, 47 (2014), pp. 1027–34.

[2] C. Bertoglio, P. Moireau, and J.-F. Gerbeau, *Sequential parameter estimation for fluid-structure problems: Application to hemodynamics*, Int. J. Numer. Meth. Biomed Engng., 28 (2012), pp. 434–455.

[3] A. Borzi, K. Ito, and K. Kunisch, *An optimal control approach to optical flow computation*, Int. J. Numer. Meth. Fluids, 40 (2002), pp. 231–240.

[4] K. Chen and D. A. Lorenz, *Image sequence interpolation based on optical flow, segmentation, and optimal control*, IEEE Transactions on Image Processing, 21 (2012), pp. 1020–1030.

[5] L. Failer, D. Meidner, and B. Vexler, *Optimal control of a linear fluid-structure interaction problem*, in preparation, (2015).

[6] D. Heitz, E. Memin, and C. Schnör, *Variational fluid flow measurements from image sequences: synopsis and perspective*, Exp Fluids, 48 (2010), pp. 369–393.

[7] M. Klinger, *Variational Methods for the Estimation of Transport Fields with Application to the Recovery of Physics-Based Optical Flows Across Boundaries*, PhD thesis, University of Heidelberg, 2015.

[8] M. Perego, A. Veneziani, and C. Vergara, *A variational approach for estimating the compliance of the cardiovascular tissue: An inverse fluid-structure interaction problem*, SIAM Journal on Scientific Computing, 33 (2011), pp. 1181–1211.

[9] T. Richter and T. Wick, *Optimal control and parameter estimation for stationary fluid-structure interaction problems*, SIAM J. Scientific Computing, (accepted 2013).

[10] T.Liu and L. Shen, *Fluid flow and optical flow*, J. Fluid Mech., 614 (2008), pp. 253–291.